ELECTRICAL SIMULATION OF FIELDS IN RELATIVE MOTION

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A description is given of an electrical model which permits simulation of problems in field theory involving relative motion of parts of the field simulated. The solution of one problem of this type is presented.

The method of electrical network analogs has found wide use in solving problems of field theory. For the study of stationary fields in isotropic media, described by the equations of Laplace $\nabla^2\varphi = 0$ or of Poisson $\nabla^2\varphi = \text{const}$, it is usual to employ resistance networks; for unsteady fields in isotropic media, described by the Fourier equation, $\nabla^2\varphi = (1/\chi)(\partial\varphi/\partial t)$—resistance-reactance networks are used, as well as resistance networks [4, 5]. The desired function $\varphi(x, y, z, t)$ is represented by electric potentials.

The quantity $\partial\varphi/\partial t$ is represented by a current, and the quantities $\partial^2\varphi/\partial x^2$, $\partial^2\varphi/\partial y^2$ and $\partial^2\varphi/\partial z^2$—by the potential differences generating this current.

However, the equations of the types named are only special cases of the general equation of field theory, which, as is known, has the following form [1, 2]:

$$K_{11}\frac{\partial^2\varphi}{\partial x^2} + K_{22}\frac{\partial^2\varphi}{\partial y^2} + K_{33}\frac{\partial^2\varphi}{\partial z^2} + (K_{23}+K_{32})\frac{\partial^2\varphi}{\partial y\partial x} + (K_{31}+K_{13})\frac{\partial^2\varphi}{\partial z\,\partial x} + (K_{12}+K_{21})\frac{\partial^2\varphi}{\partial x\,\partial y} + u_x\frac{\partial\varphi}{\partial x} + u_y\frac{\partial\varphi}{\partial y} + u_z\frac{\partial\varphi}{\partial z} + \frac{1}{\chi}\frac{\partial\varphi}{\partial t} + A = 0. \quad (1)$$

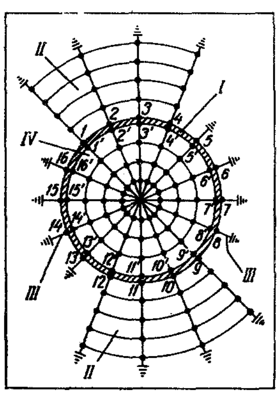

Fig. 1. Network for simulating a braking mechanism.

In this expression the coefficients K_{pq} are components of a tensor defining the anisotropic properties of the medium. From this stems the well-known method of simulating and anisotropic medium by a network analog in which resistances and capacitances in the directions of the coordinate axes must be correspondingly proportioned [3, 2]. The invariant quantity A(x, y, z) is the distributed power supplied to the simulated part of the medium. The terms $u_x(\partial\varphi/\partial x)$, $u_y(\partial\varphi/\partial y)$ and $u_z(\partial\varphi/\partial z)$ are components of the vector of mechanical displacement of the simulated part of the medium relative to another part of the medium, specifying the boundary conditions.

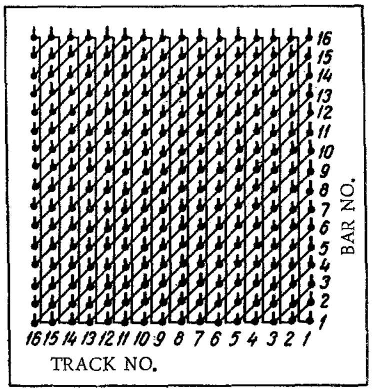

Fig. 2. Diagonal network for commutation of bars of the step-by-step switches (the vertical dashes are bars; the thin lines the wires joining them).

Problems of this type are often encountered in engineering. They are involved, for example, in the study of the distribution of heat in bearings and braking mechanisms where there is mutual slippage of friction surfaces, in heat exchangers under conditions of laminar fluid flow, in heaters with mixing, etc. They are described by an equation obtained from (1) by omitting the third to the fifth terms.

For the simulation of such processes in the analog simulation and programming laboratory of the Moscow Institute of Technology, a special electrical simulator has been developed and built. In this equipment the simulated parts of a medium are represented by RC networks of appropriate configuration, while their relative displacement is simulated with the aid of an appropriate switching circuit based on relays and step-by-step switches.

The required configuration of the parts of the medium being simulated is obtained by connecting to the ends of the elementary resistors and capacitors, led out in a definite order to plug connectors, an interchangeable plug-in panel which links the ends of these elements in conformity with the problem in hand. The plug-in panel is a network which is either soldered together from wires or obtained by photoetching in the form of a set of printed plates.

The switching circuit may simulate either cyclic rotary or cyclic reciprocating motion of one part of the medium relative to the other. An example of problems of the first type is simulation of the heating of a brake mechanism, and of the second type—simulation of a process in a heat exchanger operating on the counterflow principle.

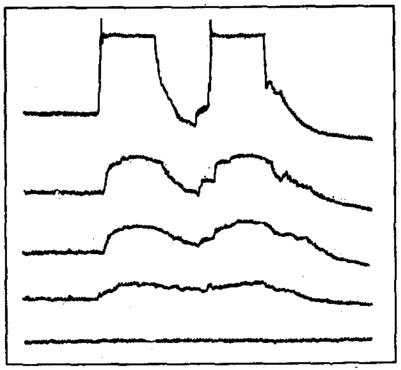

Fig. 3. Oscillogram obtained from simulation of the heat conduction equation with displacement of parts of the medium.

A model of a brake mechanism is shown in Fig. 1. The fixed part of the brake is represented by shoes II and resistors III simulating the air cooling of the brake drum IV. The drum IV is the moving part of the brake.

At fixed instants, determined by timing pulses from a special oscillator, switching occurs in switching circuit I. Whereas at the first instant points 1, 2, 3, ..., 15, 16 are connected with points 1', 2', 3', ..., 15', 16', respectively, at the second instant they are connected with 16', 1', 2', ..., 15', respectively, and then with 15', 16', 1', 2', ..., 13', 14', etc. In this way the clockwise rotation of the brake drum IV is simulated.

The switching circuit is constructed as follows. Two eight-brush step-by-step switches are controlled from a single pulse generator and may be regarded as a single sixteen-brush switch. The commutator bars of this switch are commutated in accordance with the diagonal network shown in Fig. 2. The points 1' – 16' of the network are connected to brushes with corresponding compartments, and the points 1–16— to bars with corresponding numbers.

Available step-by-step switches have 25 bars on each track, whereas, for solution of specific problems, it might be necessary to use a closed cycle with a smaller number of positions (for example, in the case under examination—16). For change-over of the simulator to different problems, a special arrangement has been provided, consisting of supplementary relays and a step-by-step switch. On receipt of a timing signal bringing the switch brushes to the last bar for a given cycle, the commutated points of the parts of the medium are disconnected from the step-by-step switches and interconnected with the aid of blocking relays. The step-by-step switches are shifted from control by the timing pulses to power feed; their brushes are returned to the original position at high speed, and then power feed is disconnected. The next timing pulse reconnects the commutated points of parts of the medium to the step-by-step switch, returning the system to the original position.

By examining the connections formed by the diagonal network, for successive location of the brushes in the first, second, etc. rows of bars, it can be directly verified that the points 1–16 and 1'–16' are commutated in precisely the order required to simulate relative rotation of the two parts of the medium.

If it is necessary to simulate relative reciprocating motion, then following return of the switches to the original position, the group of supplementary relays disconnects points 1–16 from the diagonal network and reconnects them in reverse numerical order. Thus motion in the opposite direction is simulated. After return of the switches to the original position, the group of relays also returns to the original position, and the cycle repeats.

Figure 3 shows an oscillogram, obtained on this simulator, giving the solution of the heat conduction equation for the network analog shown in Fig. 1. The oscillogram shows the temperature variation in the brake drum as a function of the angle of rotation of the drum, this being the x axis. The upper curve is temperature variation on the drum surface; the lower curves show the temperature variation at distances of (3/4)r, (1/2)r and (1/4)r, respectively, from the drum center. These curves allow one to assess the nature of temperature distribution in the brake mechanism. One can determine the temperature distribution quantitatively in the simulated field using the formula

$$\varphi = v\,\frac{R_{\varphi} q}{R_e i}, \qquad (2)$$

having first established the values of the potentials at the observation points from the calibration characteristics of the oscillograph loops.

This example was solved for a typical hoisting machine shoe brake [6], the parameters of which were changed somewhat for convenience of simulation on our simulator. According to the data of [6], this system consists of a brake drum and two shoes spanning 90° each in circumference and has the following characteristics: brake pulley diameter D_p = 300 mm; nominal braking moment M_f = 150 kgm; nominal number of revolutions of pulley per minute n = 15 rpm; the friction material is rolled strip, and the material of the pulley and shoes is 45 steel. For this problem we have λ = 40 kcal/m × hr·° C ≅ 46.52 W/m·°K; C = 0.11 kcal/kg °C ≅ 460.6 j/kg·°K; γ = 7900 kg/m^3 ≅ 76.489 kN/m^3; α = 5kcal/m·hr °C ≅ 5.815 W/m·°K.

The thermal resistance of the i-th volume element of the network in polar coordinates, in the radial direction, is determined from the formula [7]

$$R_{i\varphi} = R_{0\varphi}\,\frac{\Delta r}{r_{im}\Delta\psi\Delta l}.$$

At the edge of the pulley (i = 4) $R_{4\varphi} \cong 0.0075$ m·hr··°C/kcal ≅ 0.0066 m·°K/W. For the given network it was assumed that $R_{4e} = 10^3$ ohm.

Correspondingly, the resistors simulating the heat transfer of the pulley and the air were assumed to be $8 \cdot 10^4$ ohm.

The heat capacity of the i-th volume element was determined from the formula [6] $C_{i\varphi} = C\gamma r_m \Delta\psi \Delta l \Delta r$. On the surface of the pulley $C_{4\varphi} = 30$ kcal/°C ≅ 125.1 kg/°K. The value of the electrical capacitance at the corresponding points was 10^{-5} F.

From this we determined the time scale N_t, which, in accordance with the rate of rotation of the brake pulley at time zero, gives the rate of supply of pulses to the switching circuit:

$$N_t = \frac{t_\varphi}{t_e} = \frac{R_{i\varphi} C_{i\varphi}}{R_{ie} C_{ie}} = 2.25.$$

Knowing the friction power

$$N_f = M_f \frac{2\pi n}{60} \cong 235 \text{ kg m/sec} \cong 2305 \text{ W},$$

we can determine the heat flux

$$q = 1985 \text{ kcal/hr}.$$

The measured supply current to the simulator was 0.34 mA with a potential at the node point on the surface of 28 V; the corresponding temperature on the surface of the simulated drum was a maximum of 116° C. From the oscillogram one can find the temperatures at other times and at other points on the drum by proportion.

NOTATION

φ—temperature of simulated medium, °K; λ—thermal conductivity, W/m·°K; C—specific heat capacity, j/kg·°K; γ—specific weight, kN/m³; α—heat transfer coefficient j/kg·°K; $R_{0\varphi}$—thermal resistivity, m °K/W; Δr—mesh size in radial direction, m, r_{im}—mean radius of a layer of the network, m; $\Delta\psi$—sector angle, equal to 1; Δl—length of simulated element along z axis, equal to 1; q—heat flux, W; i-current, A; v—voltage, V; t_φ—time in system studied, sec; t_e—time in model, sec.

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